



STATISTICAL EDGE DETECTION

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Statistical Image Regularities

- 1. There are considerable statistical regularities in real images. (Field, Atick, Bialek, Ruderman, Simoncelli, Zhu, Mumford, ... Green.)
- 2 Histograms of differential filters are very similar between images.



Edge Detection

- There have been a thousand PhD theses on edge detection (computer vision myth).
- None work significantly better than Canny's master thesis (MIT 1983).

Not considering global methods such as Geman & Geman, Mumford, Osher, Zhu...

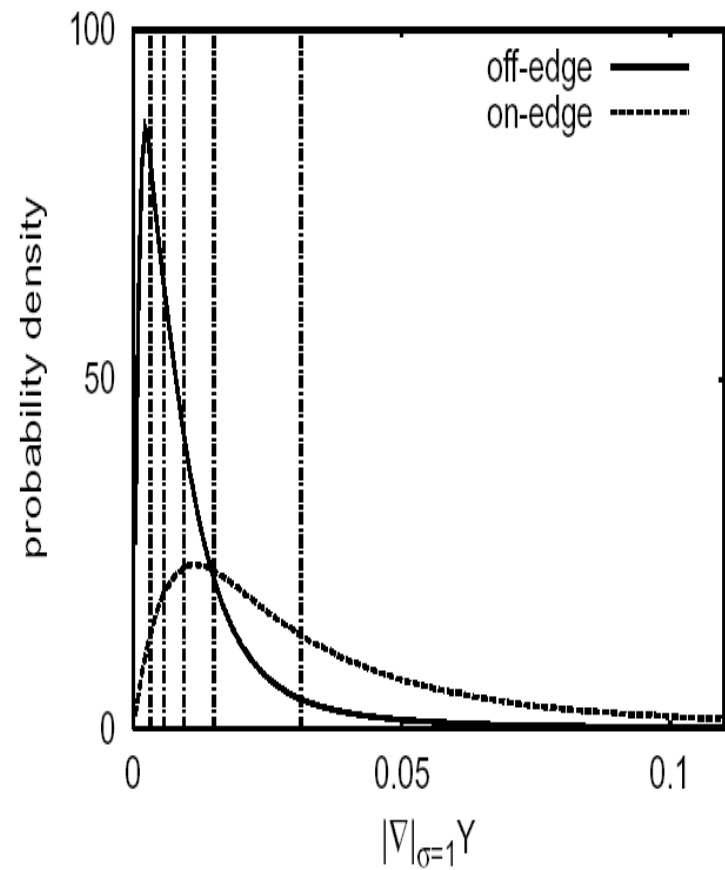


Statistical Edge Detection

- 1. Let $f(I(x))$ denote the filter response at point x on image I .
- 2. Let $P(f=y|x \text{ ON})$ and $P(f=y|x \text{ OFF})$ be the empirical distributions of the filter response, conditioned on x being ON or OFF an edge
- 3. Use loglikelihood ratio test to detect edges: $\log P(f=y|x \text{ ON})/P(f=y|x \text{ OFF}) > T$.

Example

- Let $f(I(x)) = |\text{grad } I(x)|$
- Calculate empirical histograms $P(f=y|\text{ON})$ and $P(f=y|\text{OFF})$.
- $P(f=y|\text{ON})/P(f=y|\text{OFF})$ is monotonic in y .
- So loglikelihood test is threshold on $|\text{grad } I(x)|$.





Coupling scalar filters

- Couple different edge cues by making $f(\cdot)$ vector-valued.
- Example, combine filters at different scales -- $|\text{grad } G_{\text{sig}} * I|$, where G_{sig} is a Gaussian with s.d. sig and $*$ is convolution.
- Example, combine different filters at different colour bands.



Datasets with Ground-Truth

- Sowerby Dataset – 100 colour images of English country with segmentations.

South Florida Dataset – 50 grey-scale Images with segmentations.

Berkeley Dataset – 100's of segmented images. People's judgements of edges are very similar.

Sowerby Example





Representations

- Use non-parametric representations of the histograms/probability distributions.

Problem – the number of bins increases exponentially with the dimension of the filter.

The amount of training data must grow exponentially to ensure generalization.



Example

- A 9-dim filter with 10 bins per dimension has 1,000,000,000 bins.
- But 100 images with 500 x 800 pixels (each) has approximately 2,800,000 edges (7% per image).

Not enough data.



Our Strategy

- Adapt the representation to the amount of data available. Use cross-validation to check for overlearning.
- Select histogram bin boundaries for 1-dim filter to maximize performance measures (6 bins is adequate)

Use same bins for multi-dimensional filters
AND use decision cuts (if necessary) to
Reduce the representation.



Performance Measures

- ROC curve – plot false +’ves against false -ve’s of loglikelihood test as threshold varies.
- Area under the ROC curve (error of two-alternative forced choice). Bayes risk,

Chernoff information – Bhattacharyya bound.
Motivated by theoretical studies by Yuille and Coughlan (2000).

All measures gave equivalent results.



Chernoff and Bhattacharyya

The Chernoff Information between distributions $p(y)$ and $q(y)$ is :

$$C(p, q) = -\min_{0 \leq \lambda \leq 1} \log \left\{ \sum_{y=1}^J p^{\lambda}(y) q^{1-\lambda}(y) \right\}.$$

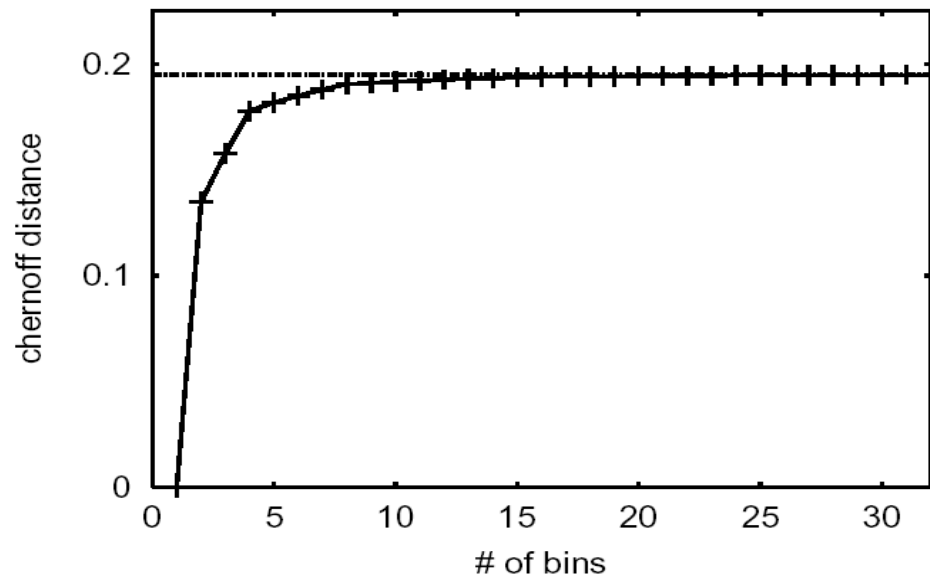
Motivated by order parameter theory for curve detection (Yuille, Coughlan 2000).

The Bhattacharyya coefficient is :

$$B(p, q) = -\log \left\{ \sum_{y=1}^J p^{1/2}(y) q^{1/2}(y) \right\}$$

Choice of 1-D Bins

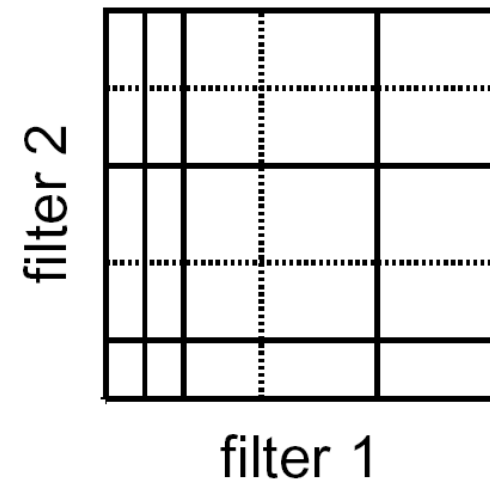
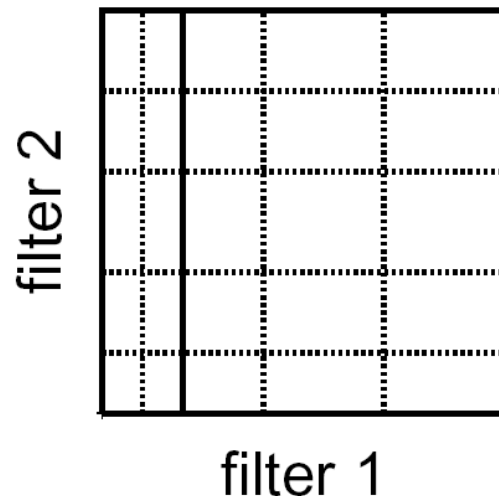
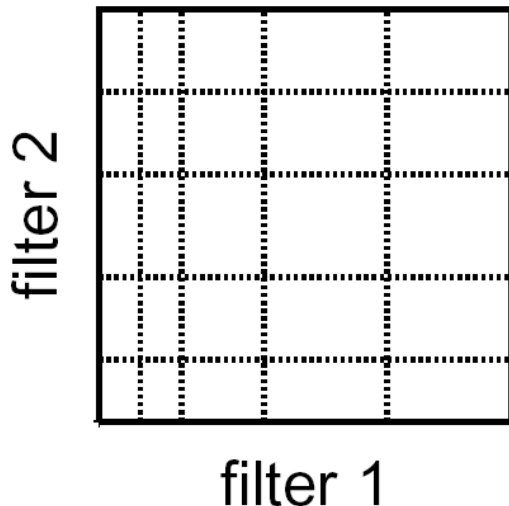
- Select bin boundaries to maximize Chernoff as a function of no. bins.
- $|\text{grad}(I)|$: $C=0.125$ for discrim. thresh.





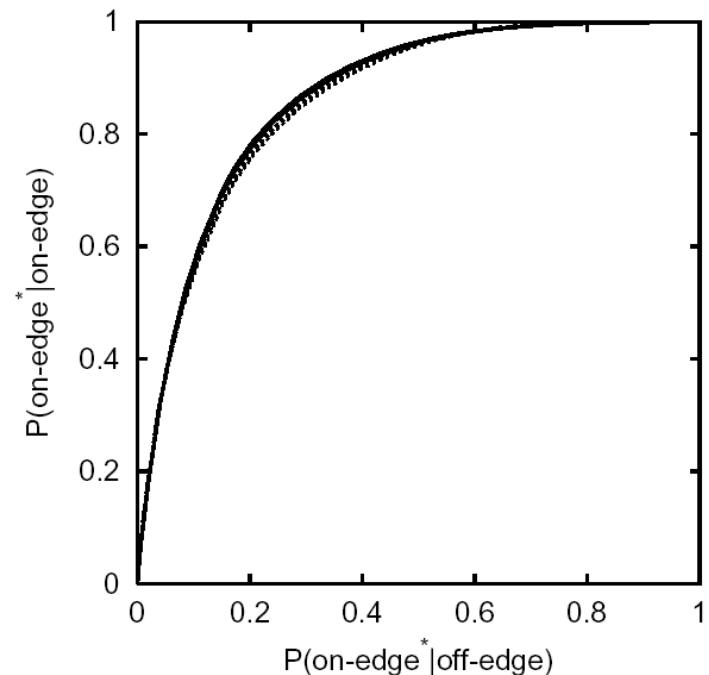
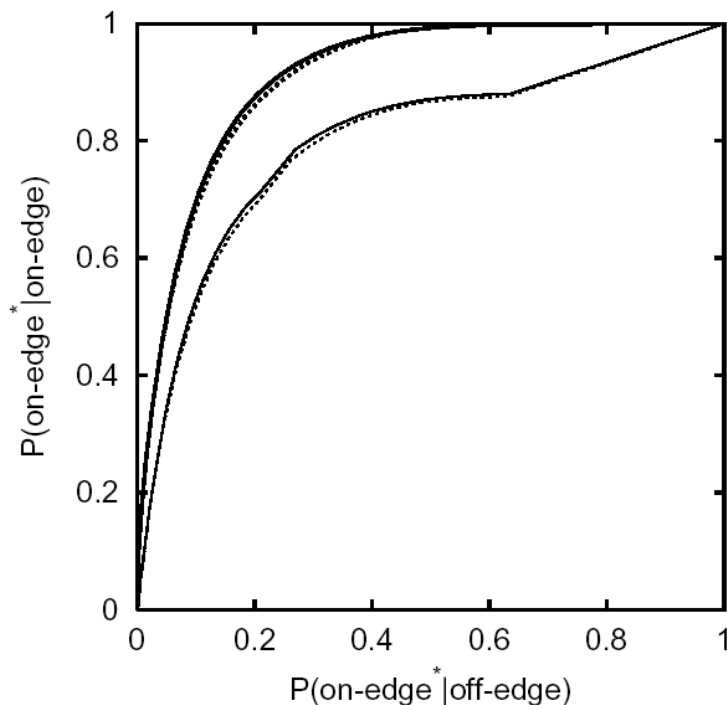
Decision Tree Representation

- Adaptively selects cuts on 1-D filter axes to maximize Chernoff. Compact representation requiring less data.



Cross-Validation

- Train on half dataset and test on rest. Overlearning (left). True learning (right).



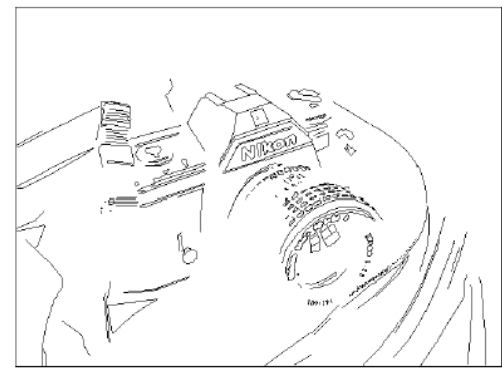
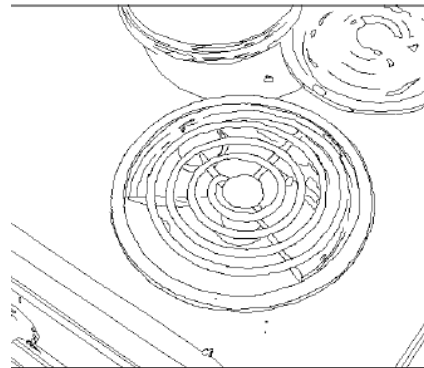
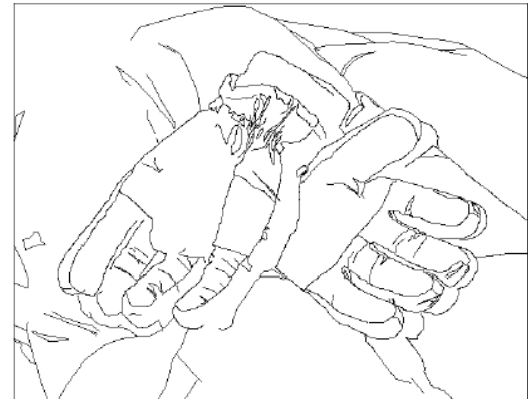
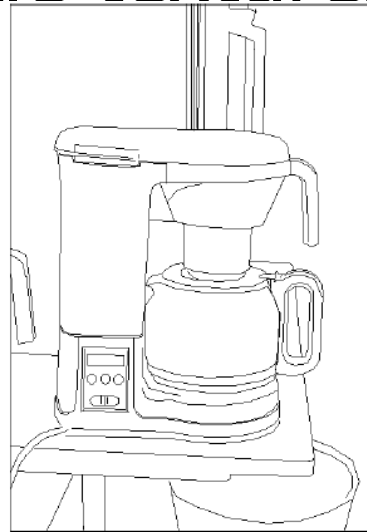
Two Datasets: I Sowerby

- Sowerby – much texture/clutter:



Two Datasets: II Florida

- South Florida: little texture/clutter





Filters

- Differential Operator: grad, Laplacian, Nitzberg, Gabors, Hilbert transform pairs.

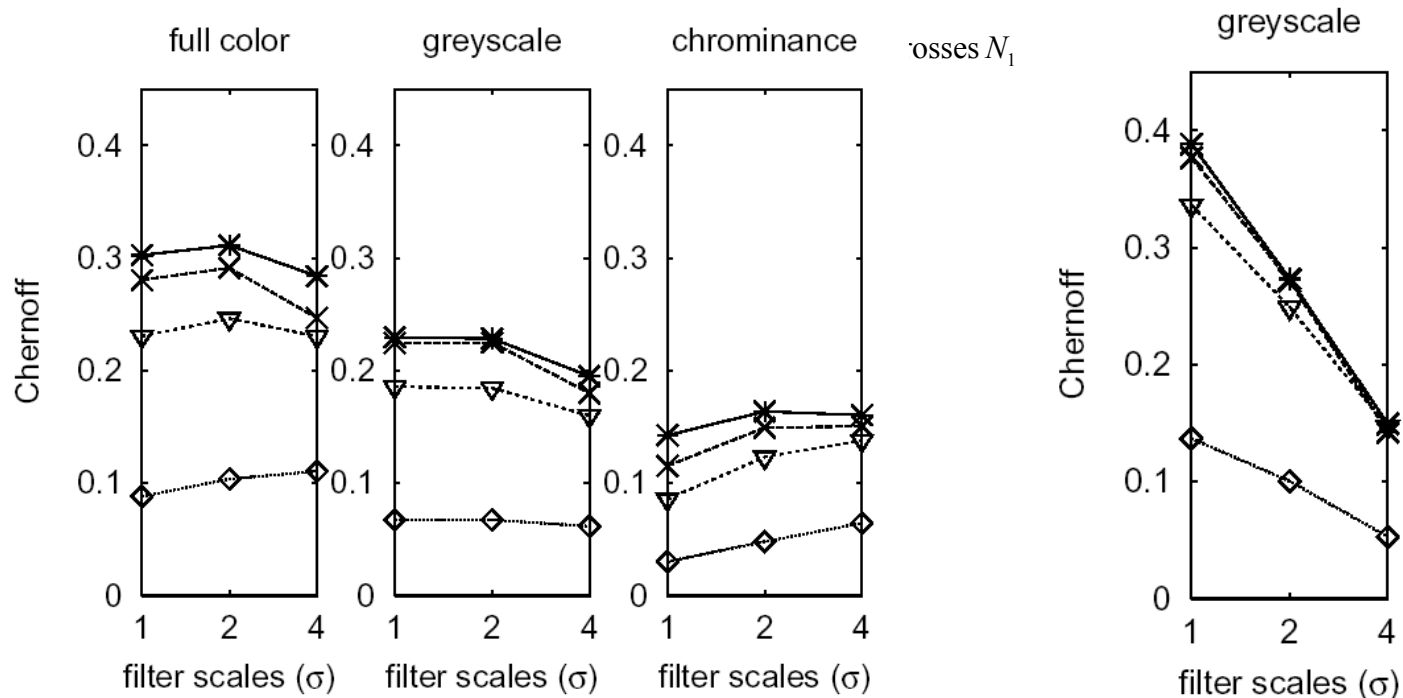
Scales: $G(\text{sig}) * I$: G Gaussian, sig SD, * Convolution.

Colour: Full colour, greyscale, chrominance

Filter Scales

- Sowerby (left), Florida (far right)

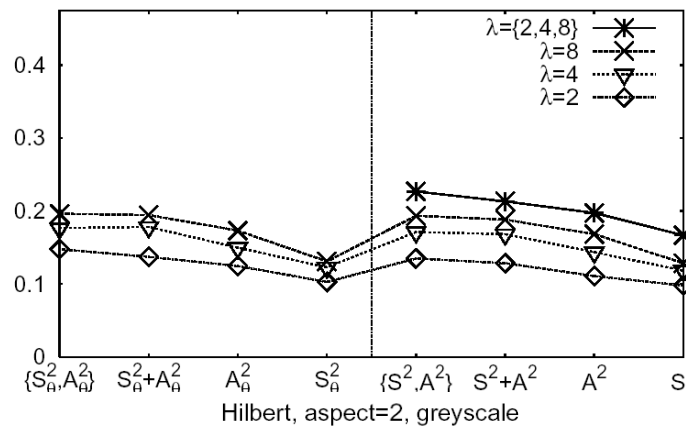
Triangles $|\nabla|$, Diamonds ∇^2 , Stars $N_{1,2}$, Crosses N_1



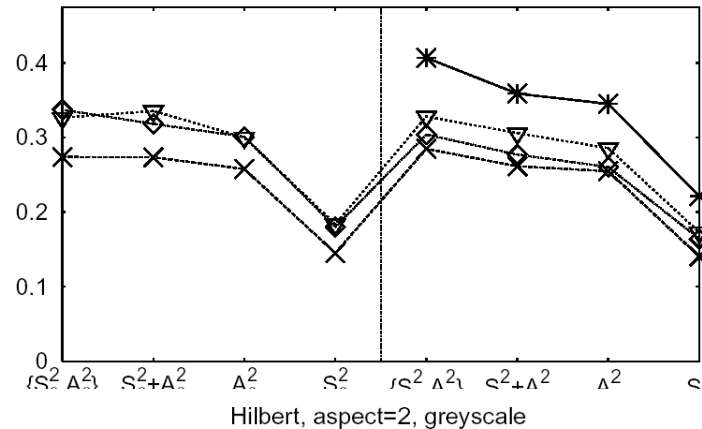
Oriented Filters: Biology?

■ Gabor filterbank/Hilbert filterbank.

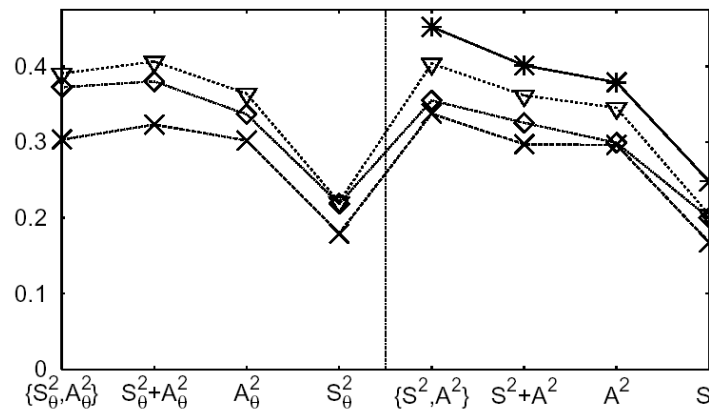
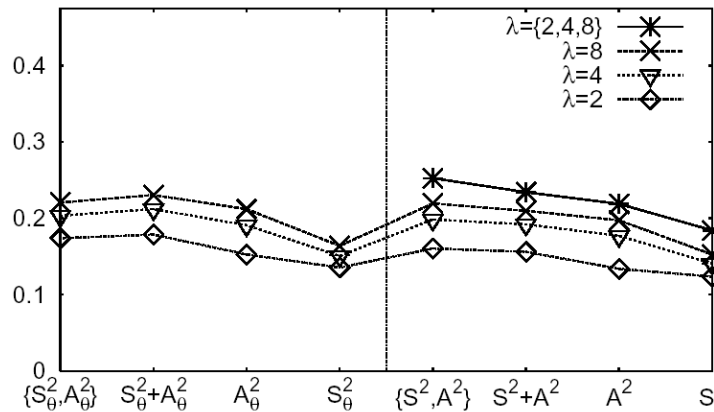
Gabor, aspect=2, greyscale



Gabor, aspect=2, greyscale



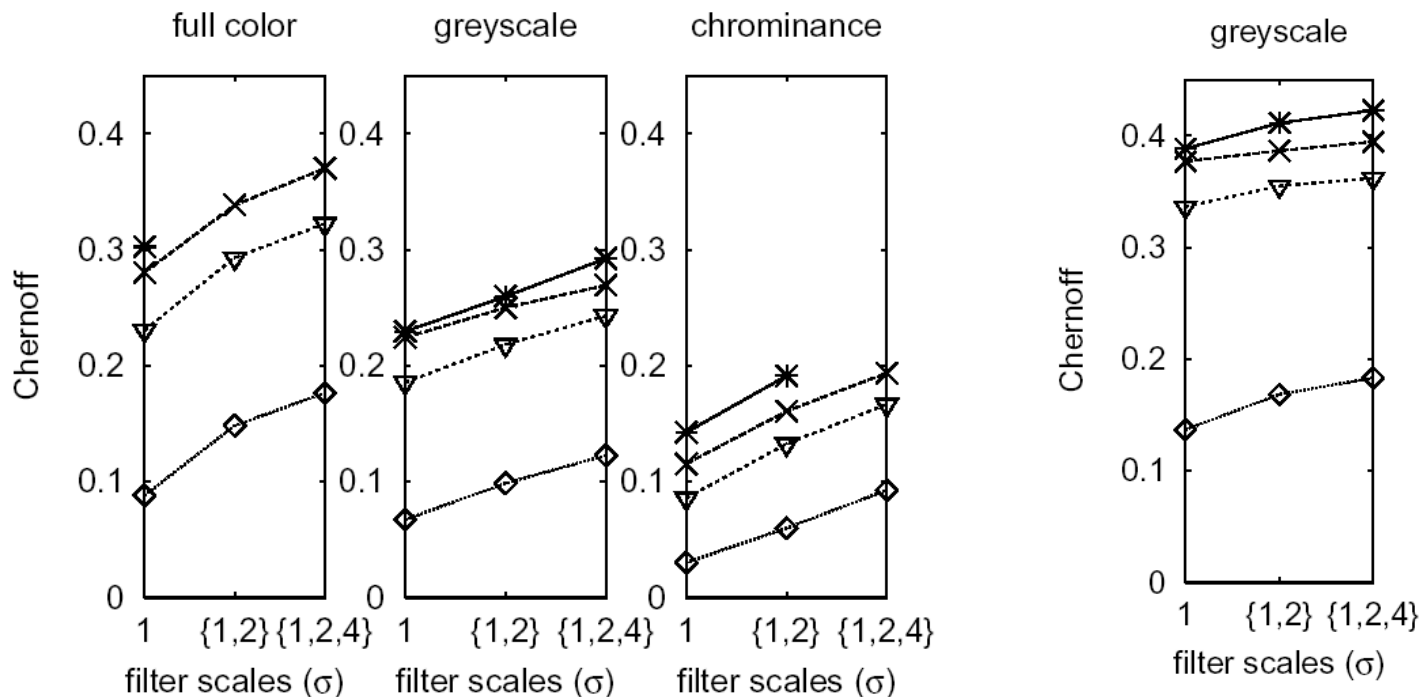
Hilbert, aspect=2, greyscale



Multiscale

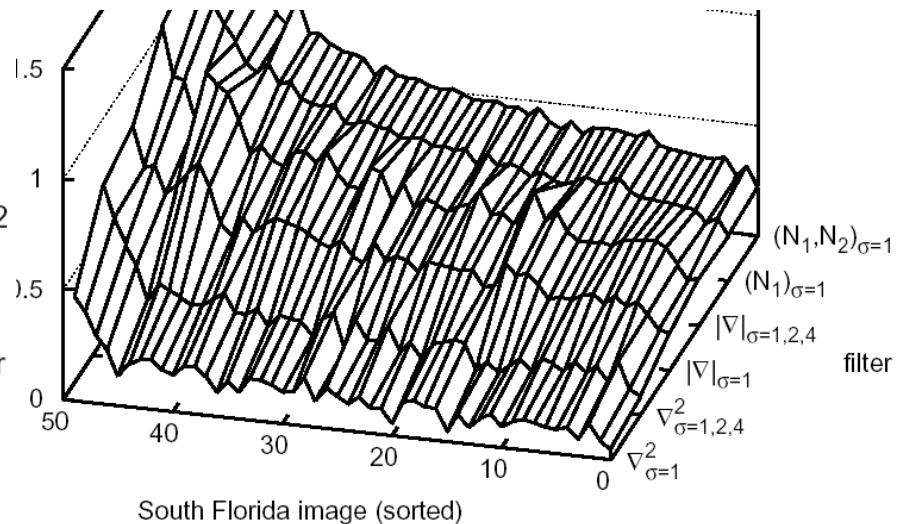
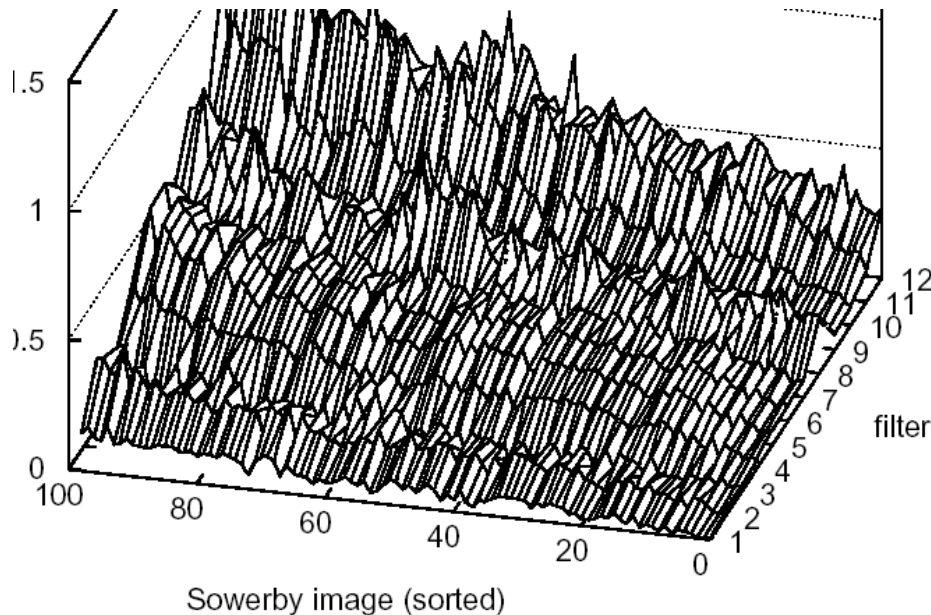
- Sowerby (left), Florida (far right)

Notation: $\{1,2\}$ – joints at scales 1,2.



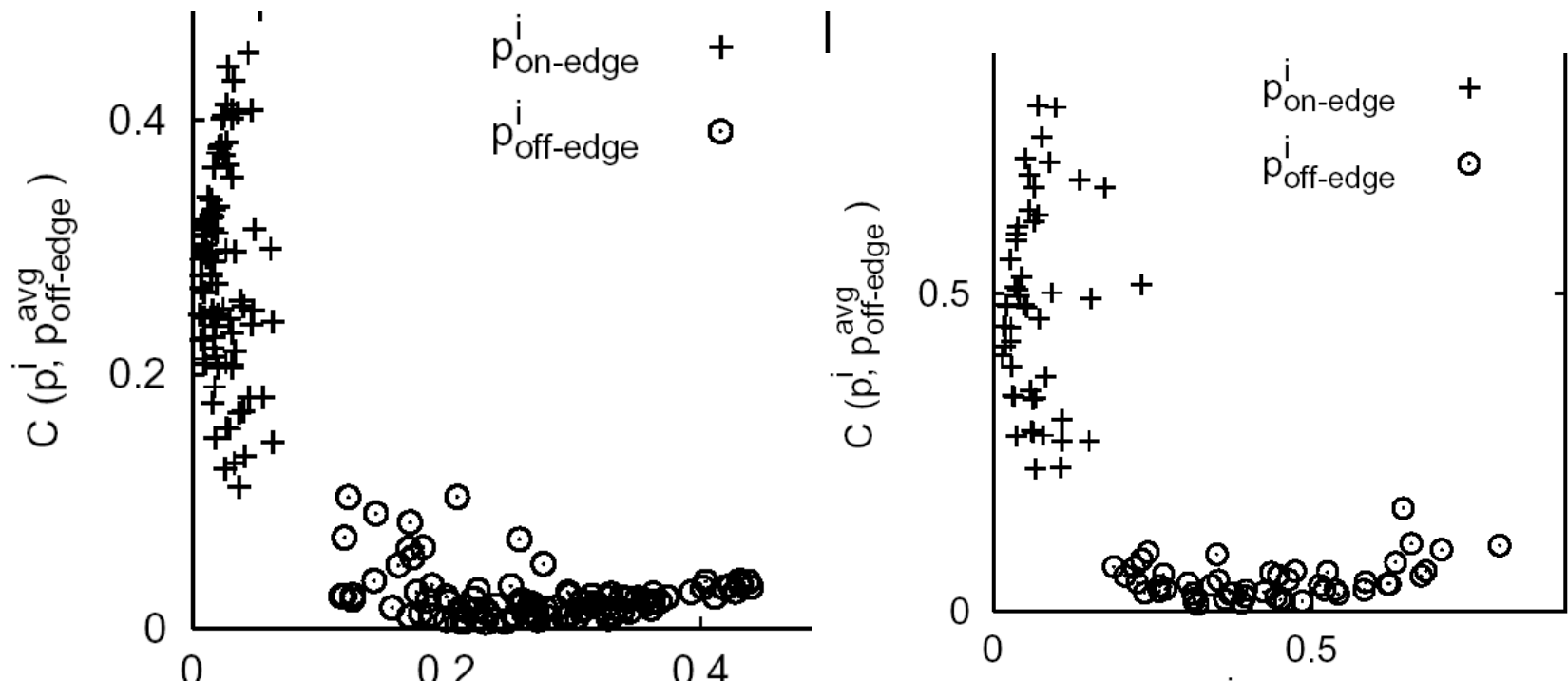
Variations between images:I

- Relative effectiveness of filter combinations is consistent over dataset.



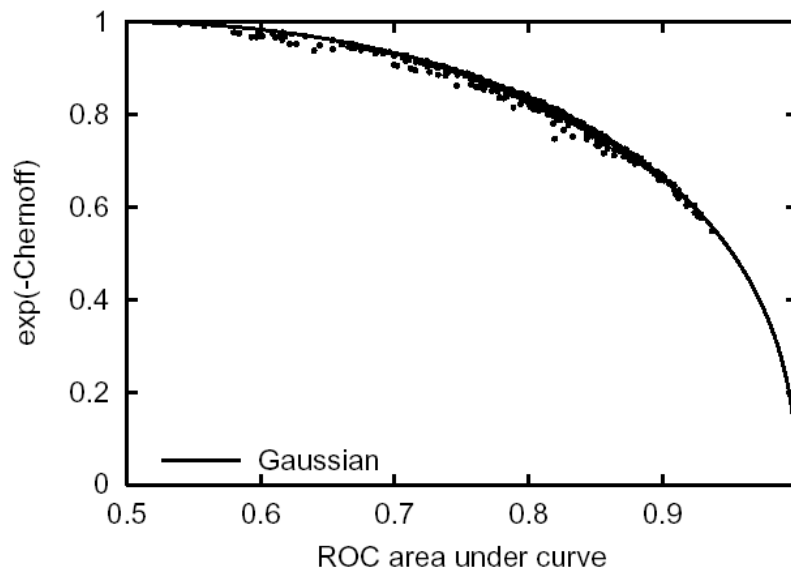
Variations between Images:II

- $P(f=y|on)$ and $P(f=y|off)$ are similar between images. Chernoffs wrt average.



Chernoff and ROC

- Conjectured relationship between Chernoff and ROC (exact for Gaussians). Induced dist. On log-likelihood.





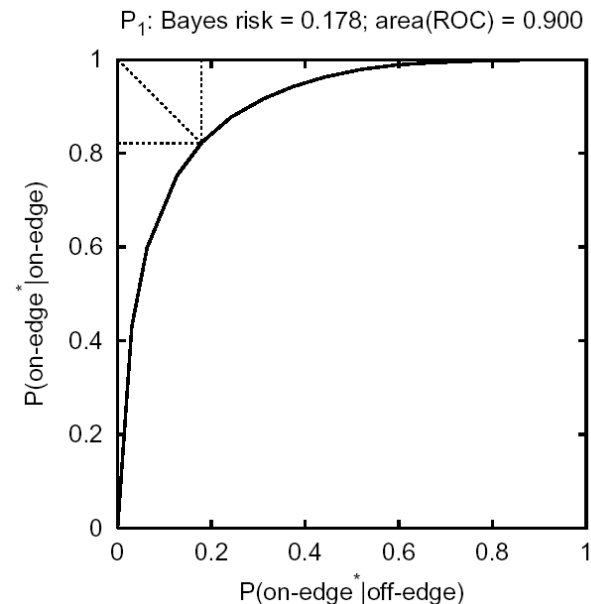
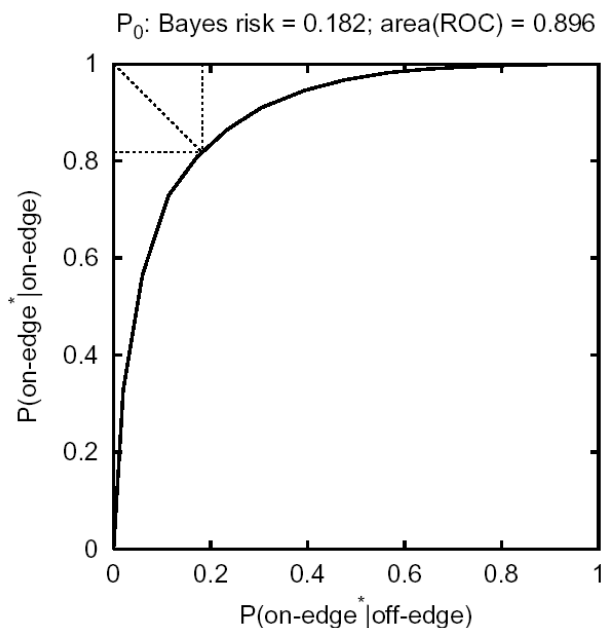
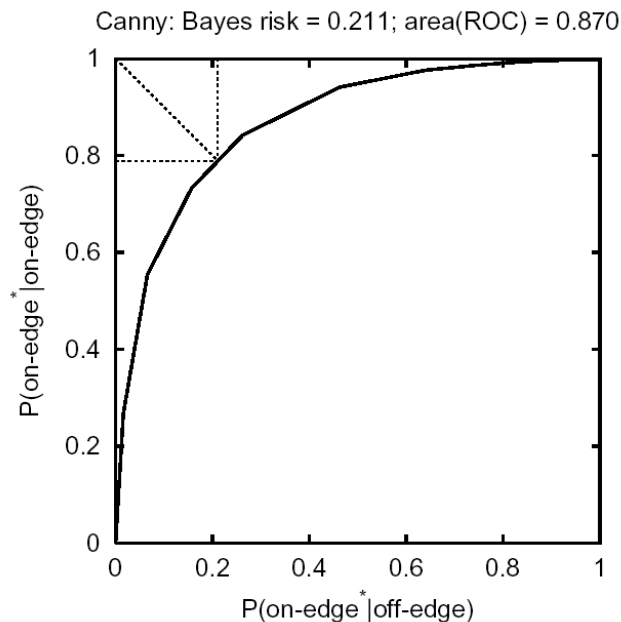
Compare w. Edge Detectors

- (I). Florida Dataset.
- Bowyer et al. (2000) evaluated 8 edge detectors. Bayes risk in range 0.035-0.045. Statistical edge detection gave Bayes risk 0.0350. Canny at 0.0352 (our implement)

Note: little texture/clutter in Florida. Edges at single scale (small scale filters most effective).

Compare w. Edge Detectors

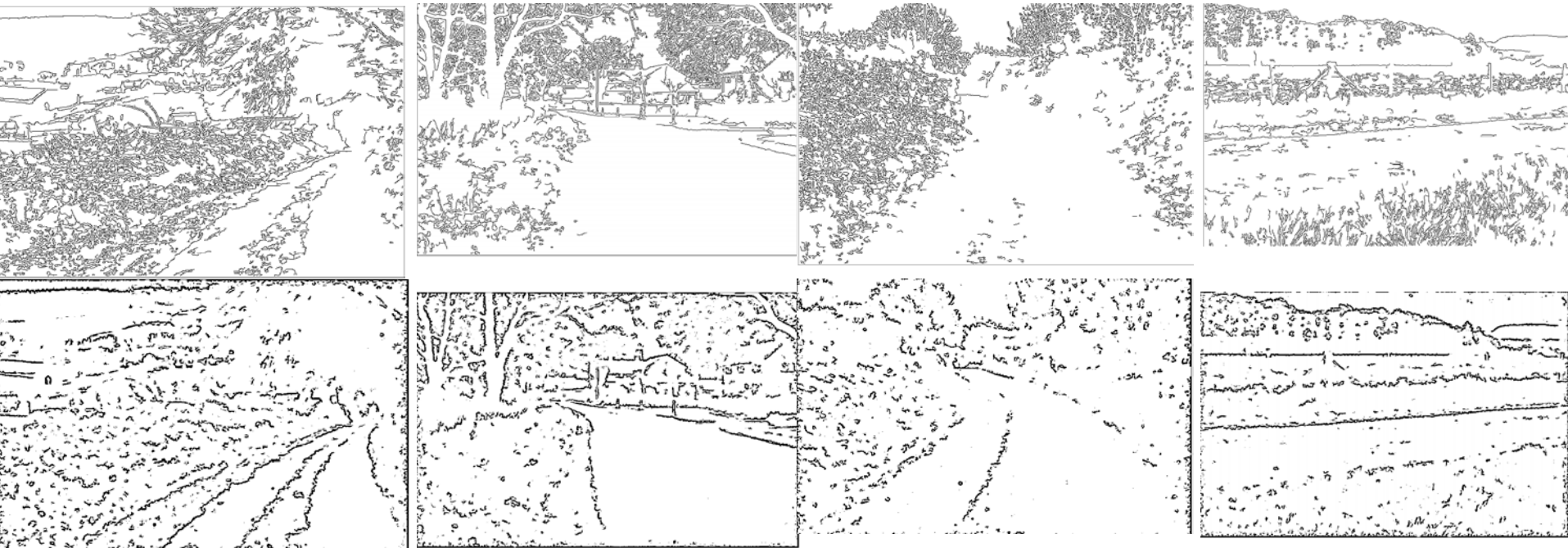
- (II) Sowerby. More texture/clutter and edges at multiple scales. Statistical edge Detection (right) outperforms Canny (left)





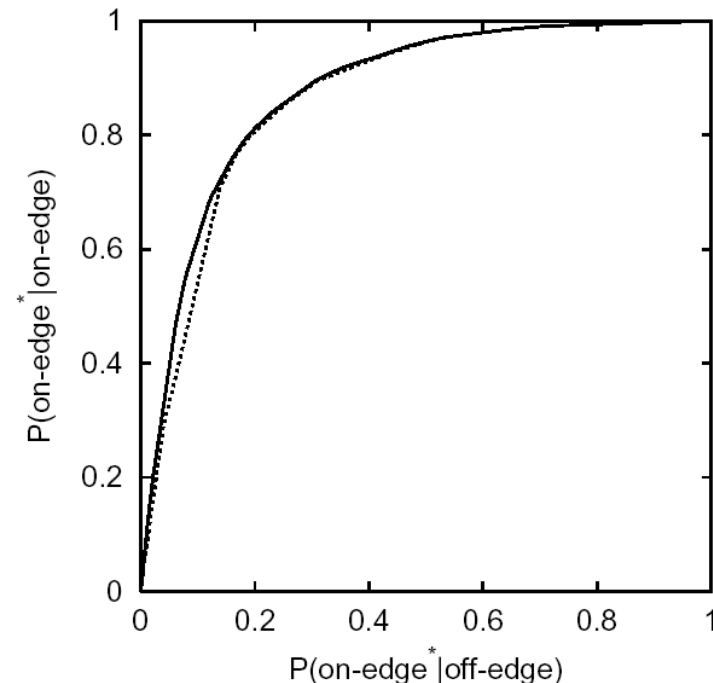
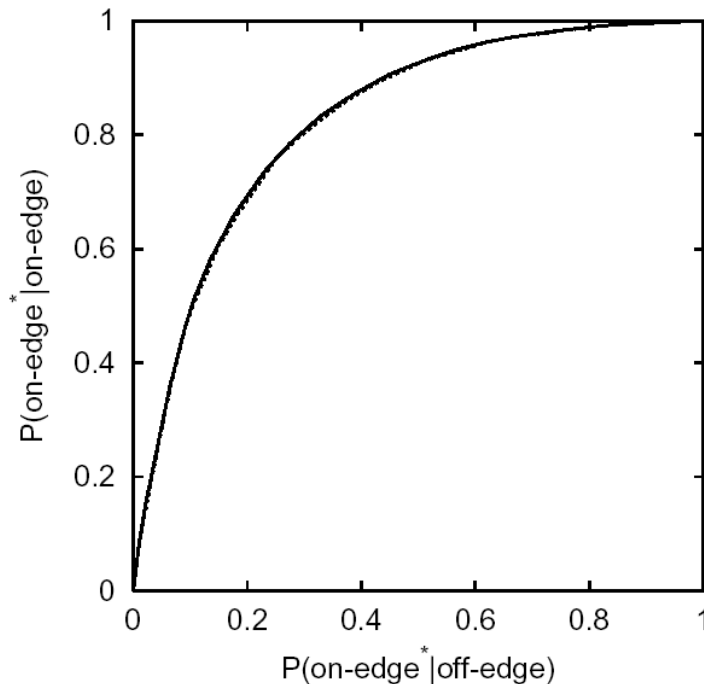
Compare w. Canny

- Canny (top), Statistics (bottom).



Adaption – Sowerby & Florida

- Learn stats on one dataset and adapt to the other. (Scaling assumption).





Extras:

- Localization: Multiple classification: on edge, 1 pixel from edge, 2 pixels, etc. (Konishi, Yuille, Coughlan 2002).

Region Identification: Vegetation, Sky, Road, Building, etc.
(Konishi and Yuille 1999).



Summary

- (I) Statistical regularities of ON and OFF edge. (Extends studies of image stats.)
- (II) Implemented a Statistical Edge Detector on 2 datasets – showed it outperformed alternatives quantitatively.
- (III) Easy to combine with other stat algs.
- (IV) There are many stat. regs. in images.